

# Dynamics of multi-modes maximum entangled coherent state over amplitude damping channel

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## Abstract

The dynamics of maximum entangled coherent state travels through an amplitude damping channel is investigated. For small values of the transmissivity rate the travelling state is very fragile to this noise channel, where it suffers from the phase flip error with high probability. The entanglement decays smoothly for larger values of the transmissivity rate and speedily for smaller values of this rate. As the number of modes increases, the travelling state over this noise channel loses its entanglement hastily. The odd and even states vanish at the same value of the field intensity.

*Keywords:* Entanglement; Quantum communication; Decoherence; Coherent states.

## 1 Introduction

Entanglement is one of the fundamental properties of quantum information theory, where it has been considered as a nonclassical resource for many applications as quantum teleportation [1] and super dense coding [2]. To achieve these tasks with high efficiency one needs maximum entangled states and perfect local operations, which are very difficult to be established in the real world. Therefore, investigating the dynamics of entanglement in the presence of imperfect circumstance is very important in the context of quantum information processing. For example, the dynamics of multipartities entanglement under the influence of decoherence is investigated in [3, 4]. The dynamics of entangled atoms interact with a deformed cavity mode is investigated by Metwally [5].

Coherent states play important roles in many fields of physics, specially in quantum technologies and quantum optics [6]. For example, two entangled coherent states are used to realize an effective quantum computation [7] and quantum teleportation [8]. Allati and et al [9] have suggested a system of three modes coherent state and used it to perform quantum teleportation. Communication via entangled coherent quantum network is investigated in [10], where it is shown that the probability of performing successful teleportation through this network depends on its size.

Entanglement properties of an optical coherent entangled state consists of two entangled modes under amplitude damping channel is discussed by Wickrt [11]. The dynamics of the GHZ state through the amplitude damping channel is investigated by Konrad et. al [4]. This motivates us to investigate the entanglement properties of a class of maximum entangled coherent states consist of three modes pass through a damping channel. Also, we study the dynamics of a multi-entangled coherent state passes through this noise channel. The effect of this channel equivalence to a photon absorption followed by a phase flip operator. The suppressing of the travelling state over this channel is discussed, where we quantified the bound entanglement of the output state as well as the survival amount of entanglement.

The paper is organized as follows: In Sec.2, we review the suggested entangled multi-modes coherent state, MMCS the amount of entanglement over a perfect environment is quantified [9].

The entanglement of the MMCS over an amplitude damping channel is investigated in Sec.3, where we quantify the bound of entanglement for a maximum entangled state consists of three modes. The dynamics of multi-modes entangled coherent state passes through the damping channel is discussed. Finally, we summarize our results in Sec.4.

## 2 Perfect environment

Entangled coherent states have been proposed as an important resource in quantum information processing, ensuring or teleporting an unknown quantum states. These sates can be written as function of the Fock state [12] as,

$$|\pm\alpha\rangle = \exp(-2|\alpha|^2) \sum_{n=0}^{\infty} \frac{(\pm\alpha)^n}{\sqrt{n!}} |n\rangle. \quad (1)$$

The coherent state can be generated from the vacuum state  $|0\rangle$ , by the displacement operator  $D(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ , where  $\hat{a}^\dagger$  and  $\hat{a}$  are bosons creation and annihilation operators respectively. Among of the properties of these states is the non-orthogonality, and the overlap of two coherent states  $|\pm\alpha\rangle$  is  $\langle\alpha|-\alpha\rangle = \exp(-2|\alpha|^2)$  which becomes orthogonal by increasing the amplitude  $|\alpha|$ . Two coherent states can be used as basis states of a logical qubit where  $|0\rangle_L = |\alpha\rangle$  and  $|1\rangle_L = |-\alpha\rangle$ . One form of the entangled coherent states between three modes can be written as,

$$\begin{aligned} \rho_\alpha = & \frac{1}{N_\theta^2} \left\{ |\sqrt{2}\alpha, \alpha, \alpha\rangle_{345} \langle\sqrt{2}\alpha, \alpha, \alpha| + e^{-i\theta} |\sqrt{2}\alpha, \alpha, \alpha\rangle_{345} \langle-\sqrt{2}\alpha, -\alpha, -\alpha| \right. \\ & + e^{i\theta} |-\sqrt{2}\alpha, -\alpha, -\alpha\rangle_{345} \langle\sqrt{2}\alpha, \alpha, \alpha| \\ & \left. + |-\sqrt{2}\alpha, -\alpha, -\alpha\rangle_{345} \langle-\sqrt{2}\alpha, -\alpha, -\alpha| \right\}, \end{aligned} \quad (2)$$

where  $N_\theta = \sqrt{2(1 + e^{-16|\alpha|^2} \cos(\theta))}$  is the normalization factor. If we set  $\theta = \pi$  in (2), one obtains a maximum entangled state defined by,

$$\rho_\alpha^- = |\psi_\alpha^-\rangle \langle\psi_\alpha^-|, \quad |\psi_\alpha^-\rangle = \frac{1}{\sqrt{N_\alpha}} (|\sqrt{2}\alpha, \alpha, \alpha\rangle - |-\sqrt{2}\alpha, -\alpha, -\alpha\rangle) \quad (3)$$

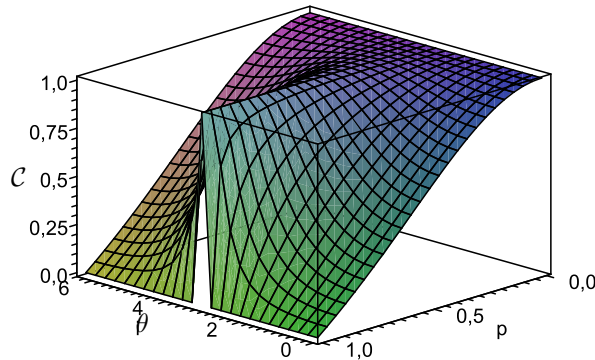


Figure 1: Concurrence for the coherent states for a range of  $\theta$  and  $p$ , with  $p = \langle\alpha|-\alpha\rangle^2$ .

where  $N_\alpha = 2(1 - e^{-8|\alpha|^2})$  is the normalization factor. We use the concurrence to quantify entanglement between two qubits, which is denoted by  $\mathcal{C}(|\psi_\alpha\rangle)$  as [14],

$$\mathcal{C}^{1/23}(|\psi_\alpha\rangle) = \frac{1 - \exp(-8|\alpha|^2)}{1 + \exp(-8|\alpha|^2)\cos(\theta)}. \quad (4)$$

Fig.(1), describes the dynamics of entanglement contained in the state  $|\psi_\alpha\rangle$  as function of  $\theta$  and  $|\alpha|$ . It is clear that, at  $\theta = \pi$  the concurrence  $\mathcal{C} = 1$  namely the entanglement is maximum and is independent of  $|\alpha|$ . However the concurrence is less than 1 ebit for the small amplitude, but it increases to one ebit for larger amplitudes. Therefore, this state represents two classes of entangled coherent states: the first is partial entangled states and the second is maximum entangled one (see [9] for more details).

### 3 Entanglement through noise environment

#### 3.1 Amplitude damping:Description

In this section we investigate the dynamics of the maximum entangled state (3), when it passes through an amplitude damping channel, which is defined by a photon loss and phase flip with probability  $p_f$ . The photon loss due to the interaction of travelling state (3) with an optical fiber prepared in a vacuum state. This interaction transfer the state  $|\pm\alpha\rangle|0\rangle_E$  to  $|\pm\sqrt{\eta}\alpha\rangle|\pm\sqrt{1-\eta}|\alpha\rangle_E$ , where  $\eta$  is called the transmissivity rate[11]. Tracing out the environment mode, one obtains a new state where the amplitude is reduced from  $\alpha$  to  $\alpha\sqrt{\eta}$  [11]. Therefore, the state vector  $|\psi_\alpha^-\rangle$  changes to  $|\psi_\eta^-\rangle$  where,

$$|\psi_\eta^-\rangle = \frac{1}{\sqrt{N_\eta}}(|\sqrt{2}\alpha\sqrt{\eta}, \alpha\sqrt{\eta}, \alpha\sqrt{\eta}\rangle - |-\sqrt{2}\alpha\sqrt{\eta}, -\alpha\sqrt{\eta}, -\alpha\sqrt{\eta}\rangle). \quad (5)$$

On the other hand, if we assume that this travelling state is subject to a phase noise with probability  $p_f$ , then the final resulting effect is equivalent to the effect of the amplitude damping channel. So, the final output state  $\rho_{adc}^-$  which is obtained from the travelling state (3) through amplitude damping channel is given by,

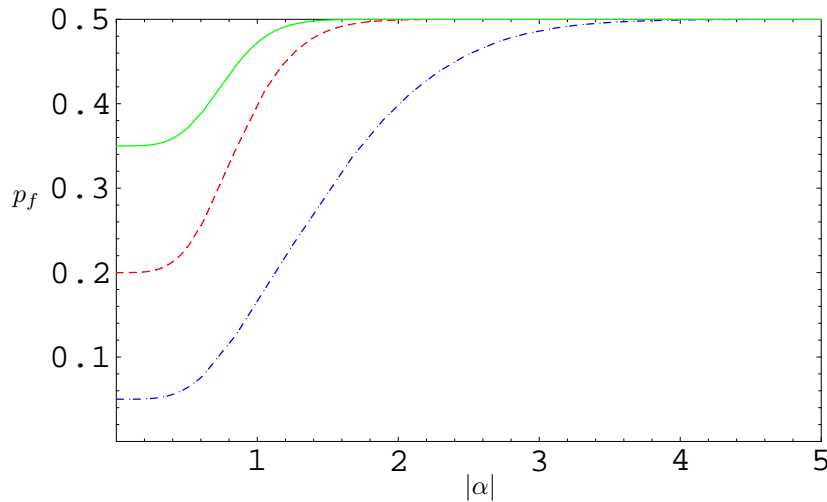


Figure 2: Phase flip probability  $p_f$  as function of the field intensity  $|\alpha|$ . The dash-dot, dot and solid curves for  $\eta = 0.9, 0.6$  and  $0.3$  respectively.

$$\rho_{adc}^- = (1 - P_f)\rho_\eta^- + P_f Z \rho_\eta^- Z, \quad (6)$$

where,  $\rho_\eta^- = |\psi_\eta^-\rangle\langle\psi_\eta^-|$  and  $Z$  is the phase flip error, which is defined as  $Z(\lambda_1|0\rangle_L + \lambda_2|1\rangle_L) = \lambda_1|0\rangle_L - \lambda_2|1\rangle_L$ . This operator effects on the travelling state with probability  $p_f$  and with  $(1 - p_f)$  the state passes safely. In terms of  $\alpha$  and  $\eta$ , the probability  $p_f$  is given by,

$$p_f = \frac{1 - e^{-8|\alpha|^2} - e^{-4(1-\eta)|\alpha|^2} + e^{-4(1+\eta)|\alpha|^2}}{2(1 - e^{-8|\alpha|^2})}. \quad (7)$$

The behavior of the probability is shown in Fig.2 for different values of the transmissivity rate,  $\eta$ . It displays that for small values of the field's intensity ( $\alpha \simeq 0$ ), the minimum values of  $p_f$  increases as the noise strength  $\eta$  decreases. In a small range of field intensity  $\alpha \in [0, 4]$ ,  $p_f$  increases faster and reaches its maximum value ( $\frac{1}{2}$ ) as the noise strength increases. However, for larger values of the field intensity the dynamics of  $p_f$  is independent of the noise strength, where  $p_f = \frac{1}{2}$ . This means that for larger values of the transmissivity rate  $\eta \simeq 1$ , the travelling state is almost maximum and its resistance to phase flip error is stronger.

### 3.2 Dynamics of entanglement:three qubit

To investigate the entanglement of a maximum entangled tripartite state ( which is defined by (3)), passes through amplitude damping channel, we consider the following situation: Let us assume that we have a source supplies a three users, Alice, Bob and Charlie with a maximum entangled state of type (3). For simplicity, it is assumed that during the transition from the source to the users, Bob and Charlie's qubit are forced to pass through amplitude damping channel. According to this suggested scenario, the dynamics of the travelling state is given by,

$$\rho_{adc} = (1 \otimes \mathcal{S}_1 \otimes \mathcal{S}_2)\rho_\alpha^-, \quad (8)$$

where  $\mathcal{S}_1$  and  $\mathcal{S}_2$  represent the damping channels which effect on Bob and Charlie's qubits respectively. For simplicity we set  $\mathcal{S}_1 = \mathcal{S}_2 = \mathcal{S}$  and rewrite the state vector  $|\psi_\alpha^-\rangle$  by using the orthogonal basis  $u$  and  $v$  defined as,

$$|\alpha\rangle = \lambda_\alpha|u\rangle + \mu_\alpha|v\rangle, \quad |-\alpha\rangle = \lambda_\alpha|u\rangle - \mu_\alpha|v\rangle, \quad (9)$$

and  $\lambda_\alpha = (\frac{1+e^{-2|\alpha|^2}}{2})^{\frac{1}{2}}$  and  $\mu_\alpha = (\frac{1-e^{-2|\alpha|^2}}{2})^{\frac{1}{2}}$ . Then the output state vector can be written as,

$$\begin{aligned} |\psi_{out}\rangle &= (1 + e^{i\theta}) \left\{ \lambda_{\sqrt{2}\alpha} \lambda_\alpha^2 |uuu\rangle + \lambda_{\sqrt{2}\alpha} \lambda_\alpha \mu_\alpha (|uvv\rangle + |vvu\rangle) + \lambda_{\sqrt{2}\alpha} \mu_\alpha^2 |uvv\rangle \right. \\ &\quad + \left. \mu_{\sqrt{2}\alpha} \lambda_\alpha^2 |vuu\rangle + \mu_{\sqrt{2}\alpha} \lambda_\alpha \mu_\alpha (|vuv\rangle + |vu u\rangle) + \mu_{\sqrt{2}\alpha} \mu_\alpha^2 |vvv\rangle \right\} \\ &\quad + (1 - e^{i\theta}) \left\{ \lambda_{\sqrt{2}\alpha} \lambda_\alpha^2 |uuu\rangle - \lambda_{\sqrt{2}\alpha} \lambda_\alpha \mu_\alpha (|uvv\rangle + |vvu\rangle) + \lambda_{\sqrt{2}\alpha} \mu_\alpha^2 |uvv\rangle \right. \\ &\quad - \left. \mu_{\sqrt{2}\alpha} \lambda_\alpha^2 |vuu\rangle + \mu_{\sqrt{2}\alpha} \lambda_\alpha \mu_\alpha (|vuv\rangle + |vu u\rangle) - \mu_{\sqrt{2}\alpha} \mu_\alpha^2 |vvv\rangle \right\}. \end{aligned} \quad (10)$$

The lower bound of entanglement of state  $\rho_{out}$  can be quantified by using a procedure described in [4]. This procedure state that the concurrence for any two qubits state  $|\zeta\rangle\langle\zeta|$  passes either in one or two sides of channels  $\mathcal{S}_1$  and  $\mathcal{S}_2$  is bounded from above in terms of the evolution of the concurrence of the maximally entangled state under either one of the one-sided channels as:

$$\mathcal{C}[(\mathcal{S}_1 \otimes \mathcal{S}_2)|\zeta\rangle\langle\zeta|] = \mathcal{C}[(\mathcal{S}_1 \otimes \mathcal{S}_2)|\phi\rangle\langle\phi|] \mathcal{C}[|\zeta\rangle\langle\zeta|], \quad (11)$$

where  $|\phi\rangle\langle\phi|$  is a maximum entangled two qubits state. For a three qubits state we use the same procedure, where we consider GHZ state represent the maximum entangled state. Therefore the concurrence of the maximum entangled state (3) is bounded from the above as [13],

$$\mathcal{C}^{23/1}[(1 \otimes S \otimes S)\rho_\alpha^-] \leq \mathcal{C}^{23/1}[(1 \otimes S \otimes S)|GHZ\rangle\langle GHZ|] \mathcal{C}^{23/1}[\rho_\alpha^-]. \quad (12)$$

To quantify the degree of entanglement of the output state  $\rho_{out} = |\psi_{out}\rangle\langle\psi_{out}|$ , we have to reexpress the GHZ in the new basis  $u$  and  $v$  as,

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|uuu\rangle + |vvv\rangle). \quad (13)$$

As a first step, we consider one side effect of the amplitude damping channel on the GHZ state. This evolution is defined as,

$$(1 \otimes 1 \otimes S)|GHZ_{uv}\rangle\langle GHZ_{uv}| = \begin{pmatrix} a & 0 & 0 & 0 & 0 & 0 & 0 & f \\ 0 & b & 0 & 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^* & 0 & 0 & 0 & 0 & c & 0 \\ f^* & 0 & 0 & 0 & 0 & 0 & 0 & d \end{pmatrix}, \quad (14)$$

where,

$$\begin{aligned} a &= P_s \frac{\lambda^2 \sqrt{\eta} u}{4\lambda_u^2}, \quad b = (1 - P_s) \frac{\mu^2 \sqrt{\eta} u}{4\mu_u^2}, \quad f = P_s \frac{\mu \sqrt{\eta} u \nu \sqrt{\eta} u}{4\mu_u \nu_u}, \\ e &= -(1 - P_s) \frac{\lambda \sqrt{\eta} u \mu \sqrt{\eta} u}{4\lambda_u \mu_u}, \quad c = (1 - P_s) \frac{\lambda^2 \sqrt{\eta} u}{4\lambda_u^2}, \quad d = P_s \frac{\mu^2 \sqrt{\eta} u}{4\mu_u^2}. \\ P_s &= \frac{1}{2} + \frac{e^{-4(1-\eta)|u|^2} - e^{-4(1+\eta)|u|^2}}{2(1 - e^{-8|u|^2})}. \end{aligned} \quad (15)$$

It is clear that, the outer and the inner elements of the state represent the "unflipped" and "flipped" GHZ states of reduced,  $\sqrt{\eta}u$  amplitude respectively. Then the dynamics of GHZ state through two-sides amplitude damping channel is given by,

$$\begin{aligned} (1 \otimes S \otimes S)|GHZ_{u,u,u}\rangle\langle GHZ_{u,u,u}| &= P_s |GHZ_{u,\sqrt{\eta}u,\sqrt{\eta}u}\rangle\langle GHZ_{u,\sqrt{\eta}u,\sqrt{\eta}u}| \\ &+ (1 - P_s) Z |GHZ_{u,\sqrt{\eta}u,\sqrt{\eta}u}\rangle\langle GHZ_{u,\sqrt{\eta}u,\sqrt{\eta}u}| Z. \end{aligned} \quad (16)$$

The concurrence of the travelling state (10) through the amplitude damping channel is given by,

$$\mathcal{C}(\rho) = 2\max[0, |e| - \sqrt{ad}, |f| - \sqrt{bc}]. \quad (17)$$

Fig.3 shows the dynamics of the concurrence  $\mathcal{C}(\rho)$  for different values of the transmissivity rate  $\eta$ . If the travelling state through the amplitude damping channel is partially entangled state i.e.  $\eta$  is small, the entanglement, which is represented by the concurrence, is very small and vanishes for small values of the field intensity. However, for larger values of  $\eta$ , the initial entanglement is large and decreases smoothly as the field's intensity increases. So, to keep the entanglement of the MMECS over the amplitude damping channel survival for a long time, one has to decrease the field's intensity. It is clear that, for larger values of the transmissivity rate  $\eta$  the travelling state is more robust.

### 3.3 Dynamics of entanglement:" $m$ modes

In this section, we assume that the users share a coherent state of  $m$  modes given by,

$$\begin{aligned} |\Psi_{0\dots m}^\pm\rangle &= A_{m+1}^\pm \left( |2^{\frac{m-1}{2}}\alpha\rangle_0 \dots |2^{\frac{1}{2}}\alpha\rangle_{m-2} |\alpha\rangle_{m-1} |\alpha\rangle_m \right. \\ &\quad \left. \pm | -2^{\frac{m-1}{2}}\alpha\rangle_0 \dots | -2^{\frac{1}{2}}\alpha\rangle_{m-2} | -\alpha\rangle_{m-1} | -\alpha\rangle_m \right), \end{aligned} \quad (18)$$

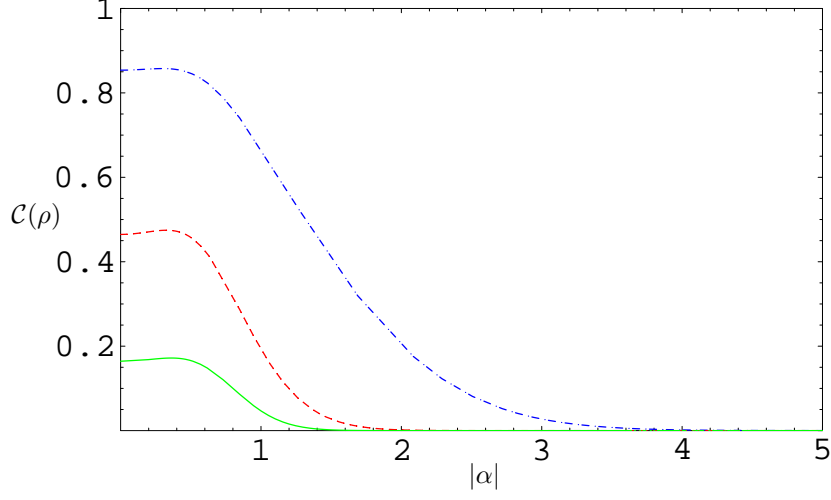


Figure 3: The concurrence  $\mathcal{C}(\rho)$  as a function of  $\eta$  and  $|\alpha|$ . The dash-dot, dot and solid curves are evaluated at  $\eta = 0.9, 0.6$  and  $0.3$  respectively.

where  $A_{m+1}^{\pm} = [2(1 \pm e^{-2^{m+1}|\alpha|^2})]^{-\frac{1}{2}}$ , is the normalized factor. This state can be generated from Schrödinger state and optics devices. In [9], we have shown that this state represents a quantum network, shared between multiusers where one user called emitter posses the mode 0 and the other users share the remaining  $m$  modes. Moreover we have employed this state to teleport a multipartite states of  $m$  modes. The degree of entanglement of the network which is defined by the state  $\rho_{gen} = |\psi_{0,\dots,m}^{\pm}\rangle\langle\psi_{0,\dots,m}^{\pm}|$  is given by [10],

$$C^{0/1,2,\dots,m} = 1. \quad (19)$$

for  $\theta = \pi$  or for  $\theta = 0$ .

The main aim of this section is investigating the entangled and separable properties of this multipartite state. Let us assume that there are  $m$  modes of the state (18) passes through an amplitude damping channel. In this case the output state can be written as,

$$\rho_{out}^{\pm} = (1 - p_{f,m})\rho_{\eta,0\dots m}^{\pm} + p_{f,m}Z\rho_{\eta,0,\dots,m}^{\pm}Z, \quad (20)$$

where,  $\rho_{out}^{\pm} = |\Psi_{0,\dots,m}^{\pm}\rangle\langle\Psi_{0,\dots,m}^{\pm}|$  and  $p_{f,m}$  is the probability that the phase flip affects the travelling state through the amplitude damping channel. This probability is given by,

$$p_{f,m} = \frac{1 - e^{-2^m|\alpha|^2} - e^{-2^{m-1}(1-\eta)|\alpha|^2} + e^{-2^{m-1}(1+\eta)|\alpha|^2}}{2(1 - e^{-2^m|\alpha|^2})}, \quad m \geq 1 \quad (21)$$

Fig.(4a), displays the dynamics of the probability  $p_f$  of the phase bit flip error which effects on the travelling state through the amplitude damping channel for different values of the modes while transmissivity rate is large ( $\eta = 0.99$ ), i.e. the travelling state is almost maximum. It is clear that, for small vales of modes, the probability  $p_{f,m}$  increases gradually to reach its maximum value ( $= 0.5$ ) for lager values of the field intensity  $|\alpha|$ . However for larger values of  $m$ ,  $p_{f,m}$  increases abruptly and reaches the maximum bound for small values of the field intensity. In Fig.(4b), we assume that the travelling state (18) through the amplitude damping channel is partially entangled state, where we set the transmissivity rate  $\eta = 0.1$ . In this case, the resistance of the input state (18) for the phase bit flip error is very fragile, where  $p_{f,n}$  reaches its maximum values for smaller values of the field intensity.

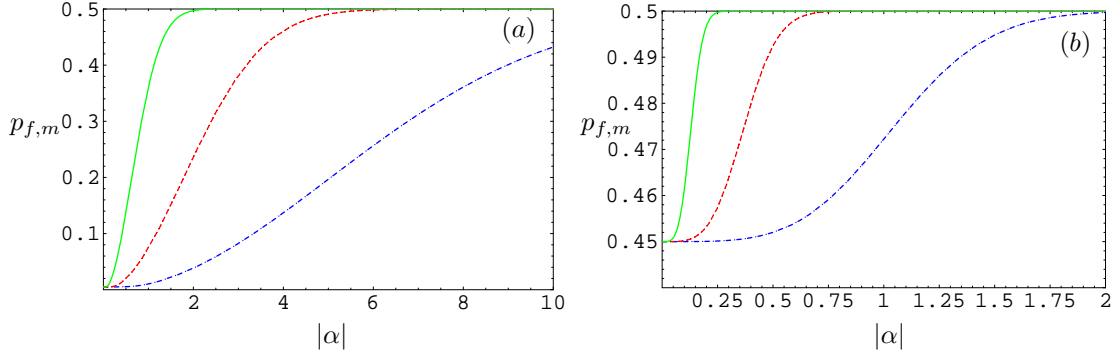


Figure 4: The probability of the phase flip error  $p_{f,m}$  which effects on the travelling state (18). The dash-dot, dot and solid curves for  $m = 2, 5, 8$  respectively and the transmissivity rate (a)  $\eta = 0.99$  and (b)  $\eta = 0.1$ .

To quantify the degree of entanglement contained in the travelling state (18) through the amplitude damping channel, we rewrite the lower bound of entanglement to include  $m$  modes. Therefore Eq.(12) can be generalized as,

$$C[(1 \otimes \dots \otimes S \otimes S)\rho_{gen}^{\pm}] \leq C[(1 \otimes \dots \otimes S \otimes S)|GHZ_{\alpha,\dots,\alpha}\rangle\langle GHZ_{\alpha,\dots,\alpha}|]C^{1/2\dots m}[\rho_{gen}^{\pm}]. \quad (22)$$

To evaluate this bound of entanglement, one has to investigate the effect of the amplitude noise channel on the  $m + 1$  modes of GHZ state which in the orthogonal basis takes the form,

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|u\dots u\rangle + |v\dots v\rangle). \quad (23)$$

The dynamics of  $|GHZ\rangle$  state through the amplitude damping channel is given by,

$$(1 \otimes \dots \otimes S \otimes S)|GHZ_{\alpha,\alpha,\dots,\alpha}\rangle\langle GHZ_{\alpha,\alpha,\dots,\alpha}| = (1 - P_{f,m})|GHZ_{\alpha,\sqrt{\eta}\alpha,\dots,\sqrt{\eta}\alpha}\rangle\langle GHZ_{\alpha,\sqrt{\eta}\alpha,\dots,\sqrt{\eta}\alpha}| + P_{f,m}Z|GHZ_{\alpha,\sqrt{\eta}\alpha,\dots,\sqrt{\eta}\alpha}\rangle\langle GHZ_{\alpha,\sqrt{\eta}\alpha,\dots,\sqrt{\eta}\alpha}|Z. \quad (24)$$

The amount of entanglement is quantified by means of the concurrence as,

$$\mathcal{C}_{\pm} = \frac{1 - 2\mathcal{P}_{f,m}}{1 \pm \exp\{-2^{m-1}(1 + \eta)|\alpha|^2\}} \sqrt{1 - \exp(-2^m|\alpha|^2)} \sqrt{1 - \exp(-2^m\eta|\alpha|^2)}, \quad (25)$$

where  $\mathcal{C}_+$  and  $\mathcal{C}_-$  for  $\theta = 0, \pi$  respectively and the concurrence  $\mathcal{C}^{1/2,\dots,m}[\rho_{gen}^{\pm}] = 1$  (see Eq.(19)).

The dynamics of entanglement which is represented by concurrence for different values of the phase  $\theta$  is described in Fig.5, where the transmissivity rate  $\eta$  is assumed to be fixed. It is clear that, for odd state i.e.  $\theta = \pi$ , the concurrence decreases as the field intensity increases as shown in Fig.(5a). The decay of entanglement depends on the number of modes of the travelling state. For small values of modes, the entanglement decays smoothly and gradually to vanishes completely at larger values of the field intensity. However as the number of modes increases the entanglement decays fast and abruptly vanishes at small values of the field intensity. The dynamics of the concurrence for an even class of MMECS, is shown in Fig.(5b), where  $\theta = 0$ . It is clear that for  $|\alpha| = 0$ , the travelling state is almost separable. However as soon as  $|\alpha|$  increases, the entanglement increases sharply to reach its maximum value in a very small range of the  $|\alpha|$  depending on the number of travelling modes. However, as  $|\alpha|$  increases more, the entanglement decays gradually

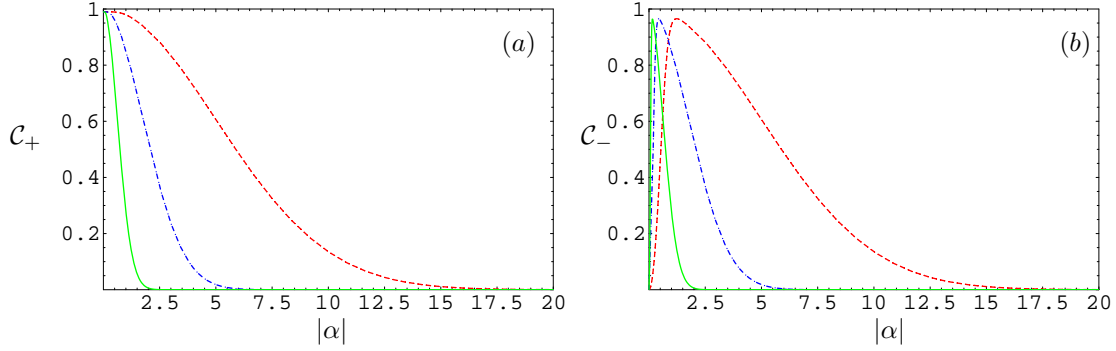


Figure 5: Dynamics of the concurrence for different values of modes. The dot, dash-dot and solid curves for  $m = 2, 5, 8$  respectively and transmissivity rate  $\eta = 0.9$  (a) For  $\theta = \pi$  (b) For  $\theta = 0$ .

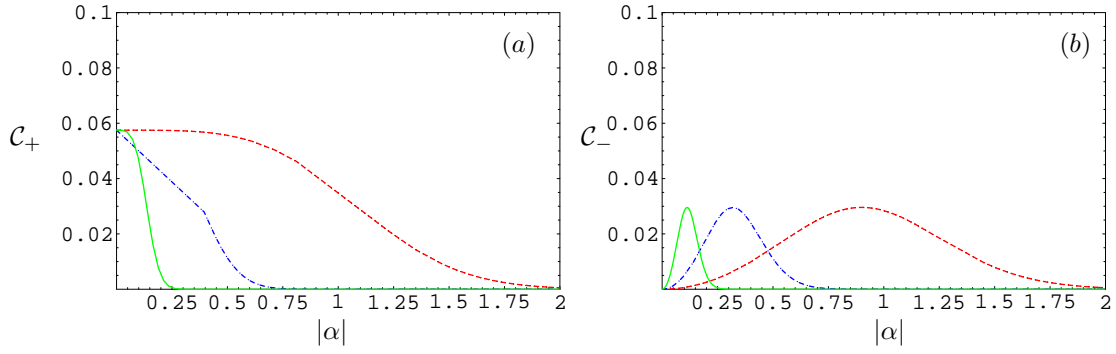


Figure 6: The same as Fig.5, but  $\eta = 0.1$ .

for small values of  $m$  and hastily for larger values of  $m$ . From Figs.(5a&5b), the entanglement vanishes for the same value of  $|\alpha|$ . Therefore, the amount of entanglement contained in the odd and even travelling states over the amplitude damping channel vanishes for the same value of the field intensity.

Fig.(6) shows the dynamics of entanglement for small value of  $\eta (= 0.1)$ , i.e the travelling state over the amplitude damping channel has an initial small value of entanglement. The general behavior is the same as that depicted in Fig.(5). However, the initial amount of entanglement is very small and vanishes very fast at small values of the field intensity.

## 4 Conclusion

The dynamics of a maximum entangled state passes through an amplitude damping channel is discussed. We showed that, the entanglement decays gradually for larger values of the field intensity and small values of the transmissivity rate. However For small values of the transmissivity rate, the entanglement vanishes at small values of the field intensity. Therefore to increase the resistance of the MMECS to entanglement degradation one has to increase the field's intensity when the transmissivity rate is large.

The dynamics of a multi-modes entangled state passes through an amplitude damping channel is investigated. This type of study displays the effect of the noise strength, the phase flip operator and the field intensity. We show that the travelling state suffering from the phase flip effect with high probability for small values the noise strength absorption parameter. However the



robustness of this multi-modes entangled state for the phase flip operator, decreases as the photon absorption decreases, where in this case the travelling state is partially entangled state. Moreover, this resistance decreases as the field's intensity increases. On the other hand, the probability of the phase error effects depends on the number of photons for each mode, where the probability is maximized as the number of photons increases.

The entanglement of MMECS for different modes is investigated, where the entanglement decreases gradually for small values of modes. However as the number of modes increases, the entanglement decays very fast for small values of the field's intensity. It is shown that the entanglement for both the odd and even MMECS states completely vanishes at the same values of the field intensity. The decay rate of the travelling entanglement depends on the field's intensity and the transmissivity rate.

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